ICHM Co-Sponsored Special Sessions on the History of Mathematics

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By Daniel Otero

This set of lectures comprised a two-day Special Session on the History of Mathematics at the Joint Meeting of the American Mathematical Society and the Mathematical Association of America held in San Diego, California, USA. The session was organized by Sloan Despeaux (Western Carolina University), Jemma Lorenat (Pitzer College), Clemency Montelle (University of Canterbury, NZ), Daniel Otero (Xavier University) and Adrian Rice (Randolph-Macon College), and featured 25 speakers from a total of six countries, including the United States and Canada. The four sessions were loosely organized by the chronological era of the subjects which the speakers treated, with talks on the early history of mathematics scheduled in the earliest session and those on twentieth century history in the latest.

The following speakers presented talks at the meeting:

Floating sexagesimal arithmetic in Antiquity (Mesopotamia, early second millennium BCE)
Christine Proust, Laboratoire SPHERE, France
Sophisticated computation methods were developed 4000 years ago in Mesopotamia in the context of scribal schools. These computation methods are based on the use of a floating sexagesimal place value notation. They rely on original notions of numbers, quantity, measurement unit, order, divisibility, algorithm, sexagesimality, etc. This presentation explores some aspects of these original notions through cuneiform texts dealing with reciprocals, factorization, and the generation of "Pythagorean" triples.

Shadow Tables and the Origins of the Tangent Function
Elizabeth Cornwall, University of Canterbury, NZ & Dixie State University, USA
Measuring shadows was of critical importance to many early cultures and were used to keep track of time, determine the altitude of the sun, among other practical applications. Their use in time keeping was particularly vital to Islamic near eastern cultures who created tables of shadows using different gnomon lengths which were included in their astronomical handbooks, or zijes. Modern scholars have found similarities between these shadow tables and our modern-day tangent and cotangent functions. Some even call them tangent tables. My talk will survey various key Arabic sources written in the $9^{th} - 13^{th}$ centuries which include tables and discussions of shadow lengths. In particular, I will examine a table of shadow lengths and investigate some of its features and the information we can draw from it. From that, I will explore some approaches modern historians have used to explain these table values and whether or not it is appropriate to label them as the tangent values resulting from the tangent function.

Ancient Greek Geographical Maps vs. Geometrical Diagrams

Jacqueline Feke, University of Waterloo, Canada

Claudius Ptolemy, the second-century mathematician, is remembered most of all for his contributions in astronomy, but just as influential was his Geography, a lengthy treatise, the majority of which consists of a catalogue of approximately eight thousand localities and their coordinates, which he intended to be drawn on a map of the known part of the world. Ptolemy deliberates on the proper structure of the map, which should maintain the proper ratios of distances between localities on the earth. Indeed, the principal aim of the Geography is the production of an image, a mathematical representation and likeness of the known part of the earth. In this talk, I investigate how Ptolemy's maps compare to the predominant type of image in the ancient Greek mathematical tradition: the geometrical diagram. I will explore the style of image, the utility, and function of ancient Greek geographical maps in contrast to geometrical diagrams.

Diagrams for Dummies: Visual Auxiliaries in printed diagrams of Euclid's Elements

Eunsoo Lee, Stanford University, USA

The printed Elements in the sixteenth century presented more concise and practical diagrams than those of previous manuscripts. While conventional diagrams were limited to implementing the description of the text, the new diagrams introduced more practical constructions absent from the text, deviating from the tradition and also from the text. This change into more practical diagrams reflects the increased emphasis on the pedagogical value of the diagram. As is evident from the compass arcs upon the diagram, readers of the Elements were invited to draw their own diagrams. This backdrop of increased engagement with the diagram facilitated learning the Elements for mathematical novices (Matheseos tyrones). These tool-based diagrams were more effective for teaching beginners than the earlier, less-functional diagrams. This paper traces a brief history of these tool-based diagrams, which I call Tyronian diagrams. Closer scrutiny is needed to determine when Tyronian diagrams first appeared and how it was circulated together with the formal version of the Elements. To this end, the paper investigates diagrams in early printed editions of the Elements in the sixteenth century. This investigation provides us with a snapshot of a key shift in diagram implementation in mathematics.

Johannes Hjelmslev and the Didactics of Geometry

Toke Lindegaard Knudsen, University of Copenhagen, Denmark

The first decades of the 20th century saw an intense discussion in Denmark of the didactic method of geometry. It became clear early on that there were only two viable paths for the teaching of geometry. One was to uphold the Euclidean ideal and teach geometry according to the axiomatic method, the other was to consider geometry as a natural science in which connections are seen through experiments. In the "experimental method," outlined in textbooks already from 1904, the pupils go as far as they can through experiments, then switch to deduce new results from the set of "axioms" brought forth by the experiments. Johannes Hjelmslev (1873–1950), who was professor of mathematics at the University of Copenhagen, considered classical geometry a crude and poor approximation to the physical world and constructed what he called "the geometry of reality" as a better model for the physical world. Some of Hjelmslev's claims, including that a tangent of a circle has a line segment in common with the circle, were rejected by some, but others took to his ideas. In particular, his followers wrote school textbooks according to his geometry. The talk will trace the discussion of the didactics of geometry in Denmark with an emphasis on the contributions by Hjelmslev.

Where Did They Learn That? Mathematical Knowledge in 18th Century Portugal

Maria Zack, Point Loma Nazarene University, USA

In 1750's, the mathematics taught at the universities in Portugal was much less advanced than what was being taught in neighboring countries. However, there were traces of sophisticated ideas from the mathematics of materials evidenced in Portuguese buildings, particularly those erected after the 1755 earthquake the leveled Lisbon. These buildings show evidence of seismic engineering. This talk discusses the role that manuscript material used in military engineering schools may have played in transmitting mathematical information from francophone Europe to Portugal. The author had been working with a little-known 1742 manuscript of a Portuguese translation of a French engineering text written by Belidor. This manuscript appears to provide one of the "missing links" in explaining how the mathematics of materials became known in Portugal well before the 1772 reformation of the mathematics curriculum taught in Portuguese universities.

Newton's Headache: the Motion of the Lunar Apse

Lawrence D'Antonio, Ramapo College, USA

Newton remarked to Halley that lunar theory gave him a headache. In particular the calculation of the motion of the lunar apse frustrated Newton (the lunar apse is an endpoint of the major axis of the ellipse defining the lunar orbit). The apse rotates approximately 3° per month due to solar perturbations, but Newton's calculations only showed half of this amount, leading Newton to say that the problem was "too complicated and cluttered with approximations." We examine the work of Newton on this problem and the later solution of Clairaut, Euler, and d'Alembert.

A Guilty Euler Searches for Large Primes

Dominic Klyve, Central Washington University, USA

Euler's first paper in number theory, written when we was 25 years old, was a tour de force of new ideas and connections in the study of primality and factoring. In the paper, he established several new lines of inquiry that he and others would spend centuries following. He also disproved the claim that Fermat numbers, integers of the form $F_n := 2^{2^n} + 1$, are all prime, and thereby removed from the mathematical world what was believed to be the easy possibility of generating arbitrarily large prime numbers. This talk will use Euler's desire to expiate his "guilt" over the factorization of F_5 as a lens to read much of his later work in number theory.

D'Alembert and the Case for Limits

Robert E. Bradley, Adelphi University, USA

Jean Le Rond d'Alembert (1717-1783) mastered the differential and integral calculus as it was practiced in Continental Europe during the first half of the 18th century and went on to introduce a number of important innovations of his own to the field. However, one of his most valuable and lasting contributions to the development of analysis was his role as an early champion of the limit concept, as opposed to the doctrine of infinitely small quantities, in providing "the true metaphysics of the differential calculus." We consider d'Alembert's arguments for this approach to the foundations of calculus, as given in Diderot's Encyclopédie and in his other writings.

Euler and Mathematical Rigor
Craig Fraser, University of Toronto, Canada
Euler is sometimes seen as a mathematician who was motivated primarily by mathematical discovery and exploration. While there were implicit assumptions and principles that informed his work, he was not a critical mathematician in the modern sense. On the other it would be inaccurate to say that he was simply naive as a mathematical thinker. There are places in his vast corpus where he explicitly considers questions of mathematical rigor, and the nature and value of proof. The paper looks at some examples and considers what we can infer from them concerning his understanding of rigor.

A Decade in the Life of William Playfair
David R Bellhouse, University of Western Ontario, Canada
William Playfair (1759 – 1823) is best known in mathematics and statistics for his invention of some graphical techniques for understanding trends in data. To provide a flavour for the range of his abilities and some of the vicissitudes of his life, I will focus on the decade 1800 – 1810. At the beginning of this decade, Playfair wrote The Statistical Breviary Which contains some of his graphs. In the middle of the decade he produced the first posthumous edition of Adam Smith's Wealth of Nations and at the end of the decade he was in the middle of publishing an enormous genealogical work, British Family Antiquity. During the same decade he was sent twice to the Fleet Prison for debt and once to Newgate Prison for his involvement in a questionable scheme to conceal information from the creditors of one of his colleagues.

The mathematical education of George Gabriel Stokes
June Barrow-Green, The Open University, UK
In 1835 George Gabriel Stokes left Ireland to study at Bristol College. From there he went up to Pembroke College, Cambridge. Coached by William Hopkins, he graduated as Senior Wrangler and first Smith's prizeman in 1841. He stayed in Cambridge for the rest of his career, being elected to the Lucasian chair in 1849, a post he held for fifty-four years until his death in 1903, his research being largely focussed on fluid mechanics and optics. In this talk I shall examine the educational environment in which Stokes' mathematical talents developed, and look at the extent to which it provided him with a platform to make a career as a mathematical physicist.

Thomas P. Kirkman – a life in mathematics
Tony Crilly, St. Albans, UK
Who was Thomas P. Kirkman (1806-1895)? Apart from fame gained from the combinatorial fifteen schoolgirls problem, what else did he do in mathematics? Just as importantly I seek to understand the conditions of his life. A lone researcher, he made incisive discoveries while serving as an Anglican minister in a remote parish in the north of England, but felt himself ignored. The paper will highlight aspects of his character, his mathematics, his philosophy and his spiritual convictions, all of which contributed to a fascinating life.

Ada's Poetic Science: Correspondences of Ada Lovelace and Charles Babbage
Gizem Karaali, Pomona College, USA
Augusta Ada, Countess of Lovelace, is today viewed as the first person to recognize the power of algorithmic machines and a pioneer in computer programming. Even though her father (the famed poet Lord Byron) abandoned her mother when Ada was a mere baby, and as a result her mother

worked tirelessly to steer her away from poetry, in Ada's life, poetry and mechanics, literature and mathematics seem to coexist peacefully. In this talk we aim to explore the interplay between mathematics and poetry in Ada Lovelace's life, and seek clues in her correspondences with Charles Babbage, the inventor of the Analytical Engine.

Left to right: Tony Crilly, Maria Zack, David Belhouse, Larry D'Antonio, Craig Fraser and Rob Bradley

What does Ada Lovelace's correspondence with Augustus De Morgan tell us about her ability?

Ursula H Martin, University of Oxford, UK
Christopher D Hollings, University of Oxford, UK
Adrian Rice, Randolph-Macon College, USA

Ada Lovelace (1815-1522) is famed as the author of a paper explaining the workings and potential of Charles Babbages's unbuilt analytical engine. She learned most of the mathematics she needed in a remarkable correspondence course that she took with Augustus De Morgan,. She worked through his textbook on differential calculus, supplemented by patching the gaps in her knowledge through more elementary textbooks. Discussions with De Morgan show her grappling with material at the frontier of current knowledge, for example divergent series, and the subtleties of Peacock's Permanence Principle. We pinpoint Lovelace's keen eye for detail, fascination with big questions, and flair for deep insights, which enabled her to challenge some deep assumptions in her teacher's work, and suggest that her ambition, in time, to do significant mathematical research was entirely credible, though sadly curtailed by her ill-health and early death.

Riemann's Twofold Path to Curvature
Paul R Wolfson, West Chester University, USA
Riemann's 1854 habilitation address leaves puzzles for historians of mathematics, because it lays out fundamental features of what we now call Riemannian geometry but offers few details of either the steps by which Riemann reached his conclusions or the insights which motivated them. Olivier Darrigol demonstrated a very plausible path directly from Gauss's work on surfaces. Others have suggested a strong connection between Riemann's physical researches and the starting point of Riemann's geometry. This talk traces a path from that starting point to Riemann's curvature via some natural geometric developments. In doing so, it explains some puzzles about his work and also something of the structure of the habilitation address.

Left to right: Mariya Boyko, Christopher Hollings and Paul Wolfson.

Dehn and Hilbert's Third Problem

John McCleary, Vassar College, USA

Among Hilbert's celebrated Paris problems, the third of the published list was the first to be solved, by Hilbert's student Max Dehn. In this presentation I will consider Dehn's solution in the context of research into the foundations of geometry that was part of Hilbert's work and the work of earlier researchers. I will consider work of Bricard and of Sforrza who are cited in Dehn's Annalen paper on the third problem. Dehn's solution will also be put into relief against the reformulations that followed on the heels of his work.

Abstraction and axioms: some parallels between 19th-century British and 20th-century American mathematics

Christopher D Hollings, University of Oxford, UK

During the nineteenth century, we see early examples of both abstract and axiomatic approaches to algebra in the works of several British mathematicians, most notably Augustus De Morgan. We also see criticisms of the associated methods. In the early decades of the twentieth century, similar ideas took a prominent place in the American mathematical community, though apparently largely independently of the prior British work. In this talk, I will look at the similarities that are present in the algebraic works of these two communities, and compare the points upon which each was criticized.

The role of socialist competition in the Soviet mathematics curriculum reform of the 1960's and 1970's.

Mariya Boyko, University of Toronto, Canada

In 1958 the Soviet government led by Nikita Khrushchev initiated a major reform of education in order to bridge the gap that then existed between the school curriculum and the practical needs of the state. Prominent mathematicians and educators (including Andrei Kolmogorov) were involved in re-writing the mathematics curriculum. However, the content of the new curriculum proved to be unsuitable for the general audience of students who were not highly interested in mathematics a priori. There are numerous academic factors that influenced such an outcome, but it is also important to explore the ideological context in which the curriculum reform was taking place. Socialist competition was one of the most prevalent ideological phenomena in the 1950's which influenced social and academic life of the state. In this talk we will focus on the role of socialist competition in the math education reform which often gets overlooked in the literature. We will define the socialist competition on international, inter-state and interpersonal level, and explore specific examples of manifestation of the socialist competition in high school and elementary school setting.

Partnership, Partition, and Proof: The Path to the Hardy–Ramanujan Partition Formula

Adrian Rice, Randolph-Macon College, USA

This year marks one hundred years since the publication of one of the most startling results in the history of mathematics: Hardy and Ramanujan's asymptotic formula for the partition function. To celebrate the centenary, this paper looks at the creation of their remarkable theorem: where it came from, how it was proved, and how the assistance of a third contributor helped to influence its ultimate form.

G.H. Hardy: mathematical biologist

Stephan Ramon Garcia, Pomona College, USA

G.H. Hardy, the great analyst who “discovered” the enigmatic Ramanujan and penned *A Mathematician’s Apology,* is most widely known outside of mathematics for his work in genetics. Hardy’s fame stems from a condescending one-page letter to the editor in *Science* concerning the stability of genotype distributions from one generation to the next. His result is now known as the Hardy–Weinberg Law, which every biology student learns today. How did Hardy, who his colleague C.P. Snow described as “the purest of the pure,” become one of the founders of modern genetics? What would Hardy say if he knew that he had earned scientific immortality for something so mathematically simple?

Charles Newton Little: America’s first knot theorist

Jim Hoste, Pitzer College, USA

Jozef Przytycki, George Washington University, USA

The modern theory of knots, a subfield of topology, arose in the latter half of the 1800s after Lord Kelvin proposed that atoms were “knotted vortices in the ether.” This led the Scottish physicist Peter Guthrie Tait to begin tabulating knots, a laborious task in which he was later joined by C.N. Little and Thomas P. Kirkman. Over a period of about 40 years, the three men created a list of all alternating knots with 11 or less crossings and all non-alternating knots to 10 crossings. While they could be sure that their tables listed, in theory, all possibilities, they had no proof whatsoever that their tables did not contain duplications. This would have to wait until well into the 20th century with the development of algebraic topology. In this talk I will review the early history of knot theory with a focus on the life and work of C.N. Little.

The editor as a scientific entrepreneur: Emile Borel and the promotion of new fields of investigation in mathematics

Caroline Ehrhardt, Universite Paris 8, France

French mathematician Emile Borel (1871-1956) was the editor of several scientific journals, book series and textbooks throughout his career. In this talk, I will focus on two of them, the “Collection de monographies sur la théorie des fonctions” and the *Revue du mois,* which he launched respectively in 1895 and 1906. In both cases, Borel used his editing function to promote new areas of research on which he worked, namely integration theory in the collection and probabilities in the Revue du mois. As he edited this two publications, Borel took advantage of his position to make cutting edge research available to a large group of readers beyond the world of mathematics. Indeed, as a series the collection offered short textbooks while the revue was a journal aimed at showcasing recent scientific work to the general public. The *Revue du Mois* deserves special emphasis, for it allow us to glimpse at the way Borel highlighted new research questions and provided explicit mathematical explanations to show that the probabilities had a role to play in the society of the early 20th century.

“Maybe from this profusion of formal logic . . . some useful idea will come”: Lebesgue, Borel, Baire, and the Birth of Descriptive Set Theory

Johann D. Gaebler, Oxford University, UK
W. Hugh Woodin, Harvard University, USA

Set theory has played a crucial role in laying the foundations of mathematics for more than a century and a half. Nevertheless, from the beginning, unique technical challenges and controversies ranging from the Burali-Forti paradox to the ubiquity of independence have beset the discipline.

We can understand many developments in set theory as attempts to respond constructively to those obstacles. In this talk, we home in on three French mathematicians at the turn of the century: Henri Lebesgue, Émile Borel, and René-Louis Baire. Their careful mediation between the traditional values of mathematical analysis and Georg Cantor’s subversive new theory secured a place for sets in the mathematical mainstream. At the same time, theirs was an uneasy truce between Cantor’s higher transfinite and what they saw as the demands of mathematics proper. Their forays into descriptive set theory shine a light on the more general trend of dephilosophication in mathematics, and how successful mathematical revolutions often travel in the guise of the establishment they replace.

Invidious Comparisons: The social and political shaping of the Fields Medal, 1936-1966

Michael J. Barany, Dartmouth College, USA

First presented in 1936, the Fields Medal quickly became one of mathematicians’ most prestigious, famous, and in some cases notorious prizes. Because its deliberations are confidential, we know very little about the early Fields Medals: how winners were selected, who else was considered, what values and priorities were debated—all these have remained locked in hidden correspondence. Until now.

My talk will analyze newly discovered letters from the 1950 and 1958 Fields Medal committees, which I claim demand a significant change to our understanding of the first three decades of medals. I will show, in particular, that the award was not considered a prize for the very best mathematicians, or even for the very best young mathematicians. Debates from those years also shed new light on how the age limit of 40 came about, and what consequences this had for the Medal and for the mathematics profession. I argue that 1966 was the turning point that set the course for the Fields Medal’s more recent meaning.